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GYRO MISALIGNMENT DECOMPOSITION APPLIED TO MESSENGER CALIBRATION*

Mark E. Pittelkau†
Daniel J. O'Shaughnessy‡

In attitude sensor misalignment estimation, a rotational misalignment vector, or a linear combination of rotational misalignments, must be constrained to zero for full observability. For this reason, one attitude sensor is generally designated the body reference sensor. Alternatively, the Inertial Measurement Unit (IMU) can be the body reference sensor. A method for removal of a rotational misalignment from a Redundant IMU (RIMU), which has more than three sense axes, was developed recently. We demonstrate the method using telemetry from the MESSENGER spacecraft. Results are compared with earlier results where one star tracker is the body reference sensor.

INTRODUCTION

The MErcury Surface, Space ENvironment, GEochemistry, and Ranging (MESSENGER) spacecraft was launched on 3 August 2004 to enter an orbit around Mercury in March 2011. The MESSENGER spacecraft carries two Galileo Avionica Autonomous Star Trackers, designated STA and STB, and a Northrop Grumman Space Inertial Reference Unit (SIRU) for attitude determination. Each star tracker provides attitude measurements at a sample rate of 10 Hz, a $16.4°$ circular field of view, and has a specified end-of-life accuracy at 0.5 deg/sec of 4.5 arcsec 1σ cross-boresight and 41 arcsec 1σ around its boresight. The present cross-boresight accuracy of the star trackers appears to be about 3.2 arcsec (cross-boresight) and 29 arcsec (boresight) for 9 tracked stars. The SIRU comprises four hemispherical resonator gyroscopes (HRGs) and provides integrated rate measurements from the four sense axes at a sample rate of 100 Hz.

In attitude sensor misalignment estimation, one rotational (absolute) misalignment vector, or a linear combination of rotational misalignments of the attitude sensors, has to be constrained to zero so that the rotational misalignment vectors are fully observable. For this reason, one attitude sensor is generally designated the body reference sensor, or "master" or "fiducial" sensor, which is not parameterized with a rotational misalignment [1]. This method was used to produce calibration results using telemetry from MESSENGER [2], where one of the star trackers was selected to be the body reference sensor.

Alternatively, the Inertial Measurement Unit (IMU) can be made the body reference sensor by removing the rotational misalignment from the gyro axis misalignments. Eliminating the rotational

†Consultant, Aerospace Control Systems LLC, 35215 Greyfriar Drive, Round Hill, Virginia 20141-2395. Tel: 540-751-1110 E-mail: mpittelkau@acsinnovations.com, URL: http://www.acsinnovations.com/. Associate Fellow AIAA, Senior Member IEEE.

‡Senior Staff, The Johns Hopkins University Applied Physics Laboratory, Laurel, MD. Tel: 240-228-3807, E-mail: daniel.oshaughnessy@jhuapl.edu

misalignment from a nominally orthogonal three-axis IMU is easy to do [3]. For a Redundant IMU (RIMU), which has more than three sense axes, elimination of the rotational misalignment is not trivial. An algorithm to separate rotational and non-orthogonal misalignments was developed [4] for a RIMU and for an IMU with three non-orthogonal sense axes. The advantages of making the IMU the body reference sensor are that it does not become occulted as can happen with star trackers, and the alignment between the IMU and payload may be more stable than between the star trackers and payload. (The star trackers on MESSENGER should never be occulted due to operational constraints, but the point stands.)

A general definition of relative misalignment was recently introduced, and a general formula for eliminating the unobservable absolute misalignment was developed [5]. This general formula imposes a constraint on the misalignments of the attitude sensors and the IMU. The weighting coefficients in the constraint matrix can be chosen such that the relative misalignments are "centered" about the average misalignment, or the coefficients can be chosen so that one of the attitude sensors or the IMU is the body reference sensor. This approach will be investigated in a sequel paper.

In this paper, we present star tracker alignment and IMU calibration estimates from the Redundant IMU Attitude Determination/Calibration (RADICAL$^{\mathrm{TM}}$) filter and telemetry from MESSENGER, and we compare the results with those reported in [2]. In these results, the IMU (the SIRU) is the body reference sensor and the star tracker misalignments are relative to the IMU. The relative misalignment between the two star trackers is computed from the misalignment relative to the IMU and compared to previous results [2].

The results in this paper are intended to illustrate the efficacy of the RADICAL calibration filter and recent advances in calibration, and to provide a *pro bono* independent analysis of the attitude sensor and gyro calibration on MESSENGER.

RADICAL ATTITUDE DETERMINATION/CALIBRATION FILTER

The RADICAL filter [6] estimates a full set of calibration parameters for $n \geq 3$ gyro axes and m attitude sensors. The filter state vector includes 3 attitude perturbation states, n gyro biases, n symmetric scale factor errors, n asymmetric scale factor errors, $2n$ gyro axis misalignments, $3m$ attitude sensor misalignments, and optionally n quantization states. The RADICAL software, which is written in C, comprises core filter functions; a command interpreter; pre-processing; data ingest, synchronization, and buffering functions; and Matlab support software for sensor simulation and for plotting and tabulating results. The core filter functions include Extended Kalman Filter functions, a command interface, telemetry interface, initialization for cold and warm start, processing of disjoint telemetry streams, default and active parameter tables, advanced measurement error models, parity residual (null space measurement) update for full observability of the gyro calibration parameters, fault detection and performance monitoring functions, diagnostic output data, telemetry output in a choice of three different size but customizable packets, and several other features. The covariance matrix in a calibration filter can become ill conditioned during its initial convergence and in other situations. Therefore UD-factorized covariance algorithms are used in RADICAL to ensure numerical stability and accuracy. The covariance matrix is never computed, except that certain elements of the covariance matrix are computed only for output and for convergence threshold tests. RADICAL is suitable for real-time on-board calibration, automated ground-based processing of telemetry, and desktop analysis and design. The RADICAL filter has been used to support NASA, military, and commercial spacecraft. The RADICAL filter was instrumental in anomaly resolution, performance verification, and provisional ground-based processing of attitude telemetry for GeoEye-1.

For ground-based processing and desktop analysis, the Command Interpreter provides a powerful and flexible command-line user interface. The Command Interpreter reads one or more script files that tell the interpreter how to execute the RADICAL filter. Commands may also be input manually or from another program, which can include commands to read script files. The script files can be nested. The Command Interpreter gives the user considerable flexibility to perform various operations during processing of telemetry data without having to modify code. The scripting commands are fairly simple in function and format. The commands define telemetry, parameter, and output file names and locations, control execution of the filter, and interaction with the filter at specified times during processing. Wildcards are defined to simplify the specification of file names.

One feature of the RADICAL calibration filter is that it can process disjoint or interrupted telemetry streams and telemetry in multiple files. This capability was demonstrated in a previous paper [2]. The attitude estimate, attitude covariance, and attitude cross-covariance are reset when there is a break in the gyro data. The parameter covariance remains intact (in UD factorized form). This is called a "warm-start" of the calibration filter. In addition, a covariance "bump" can be applied to model uncertainty due to a change in parameters since the epoch of the previously processed telemetry stream. (A covariance bump can be applied at any time during processing in RADICAL.) A bump can also be applied to the attitude covariance. The covariance bump is simply a specified increase in the covariance of any estimated parameter or attitude, and is applied upon a warm-start or at any time upon command. The bump is applied directly to the UD factors of the covariance matrix to ensure numerical accuracy and stability and for computational efficiency. The importance of being able to process disjoint telemetry streams and applying the covariance bump is that the filter does not have to be reinitialized, and the filter is nearly converged when the prior converged estimates and their covariance are used to warm-start the filter. This can be of benefit in autonomous on-board calibration. Convergence problems are avoided when a prior estimate and a small prior covariance are used to warm-start the filter, and a shorter calibration maneuver may then be sufficient to maintain convergence of the calibration parameters and their covariance. This can be of benefit during mission operations to reduce risk (for example, in the hot solar environment at Mercury), to reduce interruption of science operations, and to reduce the volume of telemetry dedicated to calibration. The warm-start feature permits a calibration maneuver to be segmented under certain operational constraints.

RIMU MISALIGNMENT VECTOR DECOMPOSITION

Let $\boldsymbol{w}_i$ be the nominal sense axis direction vectors for the gyros in a RIMU, and let the vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ form an orthogonal triad with $\boldsymbol{w}_i$ as shown in Figure 1. To measure three-axis angular rate, we have to assume that the $\boldsymbol{w}_i$ do not lie in a plane. The nonunique orthonormal vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ can be computed easily in a number of ways [1, 7]. The gyro axis misalignments are small-angle rotations $\delta_{\boldsymbol{u}_i}$ about $\boldsymbol{v}_i$ and $\delta_{\boldsymbol{v}_i}$ about $\boldsymbol{u}_i$, and the true sense axis direction vectors $\bar{\boldsymbol{w}}_i$ are modeled by small-angle rotations of the nominal $\boldsymbol{w}_i$ about the vectors $\boldsymbol{u}_i$ and $\boldsymbol{v}_i$ so that $\bar{\boldsymbol{w}}_i = \boldsymbol{w}_i - \delta_{\boldsymbol{u}i}\boldsymbol{v}_i - \delta_{\boldsymbol{v}i}\boldsymbol{u}_i$. Define the $2n \times 1$ vector of misalignments

$$\boldsymbol{\delta}_{\mathrm{g}} = \begin{bmatrix} \boldsymbol{\delta}_{\boldsymbol{u}} \\ \boldsymbol{\delta}_{\boldsymbol{v}} \end{bmatrix} \tag{1}$$

where $\boldsymbol{\delta}_{\boldsymbol{u}} = [\delta_{\boldsymbol{u}1}, \delta_{\boldsymbol{u}2}, \ldots, \delta_{\boldsymbol{u}n}]^T$ and $\boldsymbol{\delta}_{\boldsymbol{v}} = [\delta_{\boldsymbol{v}1}, \delta_{\boldsymbol{v}2}, \ldots, \delta_{\boldsymbol{v}n}]^T$. The general misalignment vector $\boldsymbol{\delta}_{\mathrm{g}}$ can be decomposed into a rotational misalignment common to all the gyro axes and nonorthogonal misalignments of the gyro axes. The rotational misalignment comprises 3 parameters, and the non-orthogonal misalignments comprise $2n - 3$ parameters. Although the conceptual

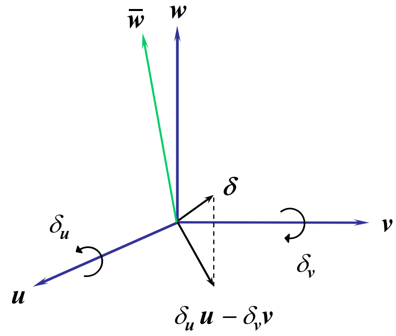


Figure 1 Gyro Axis Misalignment

meaning of the term “non-orthogonal” is a bit blurry with regard to $n \geq 3$ non-orthogonal sense axes, the term is retained due to analogy with a decomposition for 3 orthogonal sense axes [3].

The algorithm to separate rotational and non-orthogonal misalignments is summarized here. The geometry of a RIMU can be defined by the $3 \times n$ matrices $\boldsymbol{U}$, $\boldsymbol{V}$, and $\boldsymbol{W}$, where

$$\boldsymbol{U} = [\boldsymbol{u}_1 \; \boldsymbol{u}_2 \; \cdots \; \boldsymbol{u}_n] \qquad \boldsymbol{V} = [\boldsymbol{v}_1 \; \boldsymbol{v}_2 \; \cdots \; \boldsymbol{v}_n] \qquad \boldsymbol{W} = [\boldsymbol{w}_1 \; \boldsymbol{w}_2 \; \cdots \; \boldsymbol{w}_n] \tag{2}$$

Define the $2n \times 3$ matrix $\boldsymbol{Y} = [\boldsymbol{U} \;\; -\boldsymbol{V}]^T$. An orthogonal basis Q_1 for $\boldsymbol{Y}$ can be obtained from the QR factorization of $\boldsymbol{Y}$ such that

$$\boldsymbol{Y} = \boldsymbol{QR} = [Q_1 \;\; Q_2] \begin{bmatrix} R_1 \\ \boldsymbol{0} \end{bmatrix} = Q_1 R_1 \tag{3}$$

where $\boldsymbol{Q}$ is a $2n \times 2n$ orthogonal matrix partitioned into a $2n \times 3$ matrix Q_1 and a $2n \times (2n-3)$ matrix Q_2. The 3×3 upper triangular matrix R_1 is nonsingular because the $\boldsymbol{w}_i$ do not lie in a plane.

The $2n \times 1$ vector of misalignments $\boldsymbol{\delta}_\mathrm{g}$ can be expressed as a linear combination of a 3×1 vector of rotational misalignments $\boldsymbol{\delta}_\mathrm{r}$ and a $(2n-3) \times 1$ vector of non-orthogonal misalignments $\boldsymbol{\delta}_\mathrm{n}$,

$$\boldsymbol{\delta}_\mathrm{g} = [\boldsymbol{Y} \;\; Q_2] \begin{bmatrix} \boldsymbol{\delta}_\mathrm{r} \\ \boldsymbol{\delta}_\mathrm{n} \end{bmatrix} \tag{4}$$

The rotational and nonorthogonal misalignments can be computed from the general misalignment vector by

$$\begin{bmatrix} \boldsymbol{\delta}_\mathrm{r} \\ \boldsymbol{\delta}_\mathrm{n} \end{bmatrix} = [\boldsymbol{Y} \;\; Q_2]^{-1} \boldsymbol{\delta}_\mathrm{g} = \begin{bmatrix} \boldsymbol{Y}^\dagger \\ Q_2^T \end{bmatrix} \boldsymbol{\delta}_\mathrm{g} \tag{5}$$

where $\boldsymbol{Y}^\dagger = R_1^{-1} Q_1^T$ is the pseudoinverse of $\boldsymbol{Y}$.

The RADICAL calibration filter estimates the full misalignment vector $\boldsymbol{\delta}_\mathrm{g}$. If we want to make the RIMU the body-reference sensor, we have to eliminate the rotational misalignment from $\boldsymbol{\delta}_\mathrm{g}$. The rotational misalignment $\boldsymbol{\delta}_\mathrm{r}$ of the RIMU can be explicitly eliminated from the calibration model by using Eq. (4). However, this leads to a computationally less efficient filter. We can implicitly eliminate $\boldsymbol{\delta}_\mathrm{r}$ from the model by setting the initial covariance of $\hat{\boldsymbol{\delta}}_\mathrm{r}$ to a small value such as 0.001 arcsec (ideally zero but for numerical problems that would occur in the covariance update in a Kalman filter) and by setting the initial estimate of $\hat{\boldsymbol{\delta}}_\mathrm{r}$ to zero. (Estimates of a quantity are denoted by a caret.) Then a linear combination of $\hat{\boldsymbol{\delta}}_{\mathrm{g}}$, namely $\hat{\boldsymbol{\delta}}_{\mathrm{r}} = \boldsymbol{Y}^{\dagger}\hat{\boldsymbol{\delta}}_{\mathrm{g}}$, will remain close to zero during the estimation process. The covariance of the initial estimate of the general misalignment parameter vector, in terms of the covariance of the rotational and nonorthogonal misalignments, is

$$\begin{aligned} \mathrm{cov}(\hat{\boldsymbol{\delta}}_{\mathrm{g}}) = P_{\mathrm{g}} &= [\boldsymbol{Y}\ \ Q_2] \begin{bmatrix} P_{\mathrm{r}} & \mathbf{0} \\ \mathbf{0} & P_{\mathrm{n}} \end{bmatrix} [\boldsymbol{Y}\ \ Q_2]^T \\ &= \boldsymbol{Y} P_{\mathrm{r}} \boldsymbol{Y}^T + Q_2 P_{\mathrm{n}} Q_2^T \end{aligned} \tag{6}$$

where P_{r} and P_{n} are the covariance of error in the initial estimate of the rotational and nonorthogonal misalignment parameter vectors $\boldsymbol{\delta}_{\mathrm{r}}$ and $\boldsymbol{\delta}_{\mathrm{n}}$. For lack of better information, we have assumed that errors in the initial estimate of these vectors are uncorrelated, and so P_{r} and P_{n} are generally diagonal matrices.

Given a covariance matrix P_{g} of $\hat{\boldsymbol{\delta}}_{\mathrm{g}}$, we can compute the covariance and cross covariance of P_{r} and P_{n} by

$$\mathrm{cov}\left(\begin{bmatrix} \hat{\boldsymbol{\delta}}_{\mathrm{r}} \\ \hat{\boldsymbol{\delta}}_{\mathrm{n}} \end{bmatrix} \right) = \begin{bmatrix} P_{\mathrm{r}} & P_{\mathrm{rn}} \\ P_{\mathrm{nr}} & P_{\mathrm{n}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}^{\dagger} \\ Q_2^T \end{bmatrix} P_{\mathrm{g}} \begin{bmatrix} \boldsymbol{Y}^{\dagger} \\ Q_2^T \end{bmatrix}^T \tag{7}$$

RELATIVE MISALIGNMENTS

The misalignment of STB relative to STA was estimated in a previous analysis of MESSENGER telemetry [2], where STA was the body reference sensor. Here, we make the IMU the body reference sensor and we estimate the misalignment of STA and STB relative to the IMU. The misalignment of STB relative to STA, in the STB frame, is given by

$$\boldsymbol{\delta}_{\mathrm{B/A}} = \boldsymbol{\delta}_{\mathrm{B}} - \mathbf{T}_{\mathrm{STA}}^{\mathrm{STB}} \boldsymbol{\delta}_{\mathrm{A}} + \frac{1}{2} \boldsymbol{\delta}_{\mathrm{B}} \times \mathbf{T}_{\mathrm{STA}}^{\mathrm{STB}} \boldsymbol{\delta}_{\mathrm{A}} \tag{8}$$

where $\boldsymbol{\delta}_{\mathrm{A}}$ and $\boldsymbol{\delta}_{\mathrm{B}}$ are the misalignments of STA and STB relative to the IMU and are expressed in the frames of STA and STB, respectively; frame notation is only partially shown in Eq. (8). Define

$$\boldsymbol{\delta} = \begin{bmatrix} \boldsymbol{\delta}_{\mathrm{A}} \\ \boldsymbol{\delta}_{\mathrm{B}} \end{bmatrix} \qquad \text{and} \qquad P_{\boldsymbol{\delta}} = \mathrm{cov}(\hat{\boldsymbol{\delta}}) \tag{9}$$

The covariance of the relative misalignment is given by

$$\mathrm{cov}(\boldsymbol{\delta}_{\mathrm{B/A}}) = [-\mathbf{T}_{\mathrm{STA}}^{\mathrm{STB}}\ \ I]\, P_{\boldsymbol{\delta}}\, [-\mathbf{T}_{\mathrm{STA}}^{\mathrm{STB}}\ \ I]^T \tag{10}$$

where $\mathbf{T}_{\mathrm{STA}}^{\mathrm{STB}}$ is the transformation from the STA frame to the STB frame. Equation (8) is only a second-order approximation, which is sufficient for computing the covariance, but is not sufficiently accurate for computing the relative misalignment vector when the misalignments are large.

It should be noted that the nominal IMU geometry matrix, which is from the IMU vendor's calibration, contains a small rotational misalignment. Ths misalignment is not included in the estimated $\boldsymbol{\delta}_{\mathrm{A}}$ and $\boldsymbol{\delta}_{\mathrm{B}}$, although variations in the rotational misalignment of the IMU will appear in $\boldsymbol{\delta}_{\mathrm{A}}$ and $\boldsymbol{\delta}_{\mathrm{B}}$. The rotational misalignment in the nominal IMU geometry matrix is inconsequential in computing the relative misalignment of the star trackers.

CALIBRATION MANEUVER

The calibration maneuver is designed so that the calibration parameters are distinguishable in the measurements. The maneuvers are quite restricted because of thermal requirements. Inside of 0.85 AU the y-axis of the spacecraft must be within 12 degrees of the direction to the Sun. The actual calibration maneuver from 2006216 (year 2006 and day-of-year 216), where the spacecraft is within 0.85 AU, is shown in Figure 2a. Calibration maneuvers on 2005300 and 2005301 are nearly the same and so are not shown. These maneuver sequences are ± 10 deg about the x-axis, ± 10 deg about the z-axis, and ± 360 deg about the y-axis. The ± 360 deg rotation about the z-axis makes the calibration maneuver excessively long. It will be seen in the results that the calibration parameters have mostly converged in 60 min, just after the y-axis angular rate changes sign. A shorter calibration maneuver, shown in Figure 2b, was performed on 2007262. As will be seen, this maneuver is almost as effective as the longer maneuver in terms of parameter estimation accuracy. Both of these maneuvers step and settle to ± 0.3 deg/sec. For sinusoidal maneuvers, it can be shown that doubling the angle of rotation decreases the parameter estimation error by $1/\sqrt{2}$ whereas doubling the angular rate decreases the parameter estimation error by 1/2. However, the period of the maneuver is important also. A similar result should hold for piecewise-constant maneuvers.

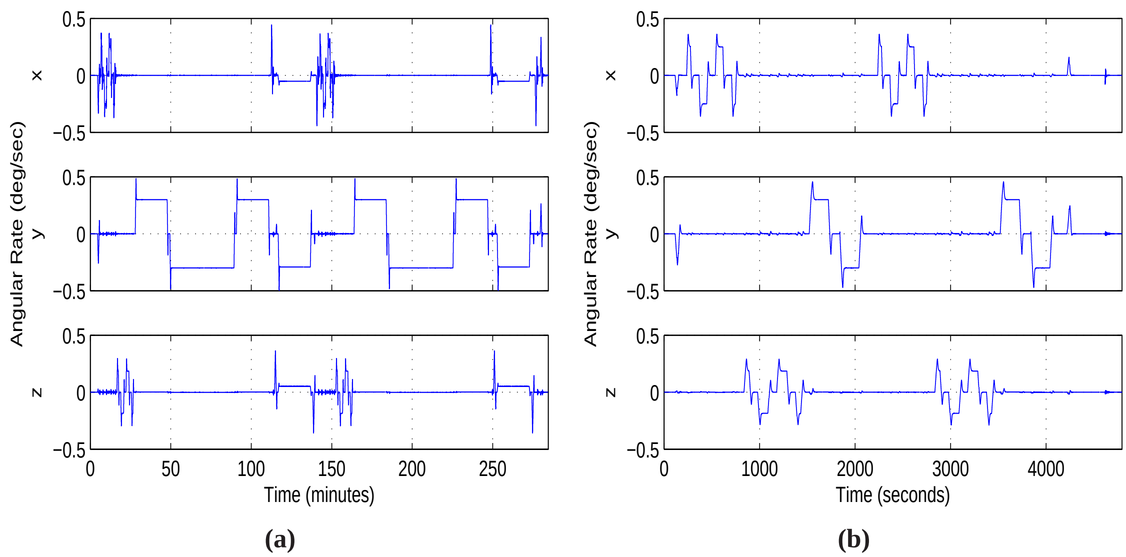


Figure 2 (a) Calibration maneuvers on 2005300, 2005301, 2006216, and (b) 2007262.

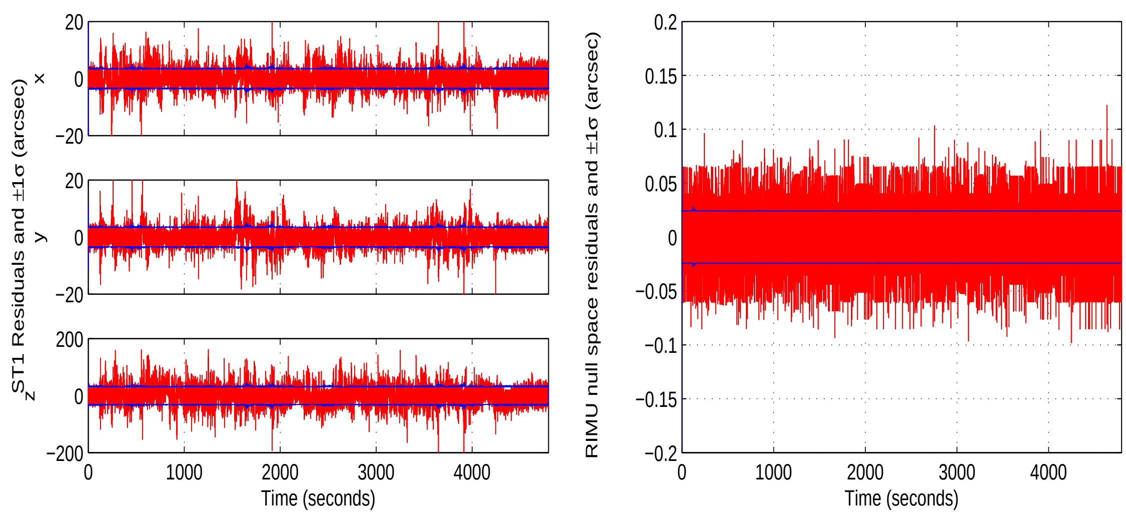
CALIBRATION RESULTS

The initial misalignment estimates are zero and are assumed to be uncorrelated. The standard deviation of error in the initial estimate of the star tracker misalignment is 1800 arcsec per axis for each star tracker. The standard deviation of error in the initial estimate $\hat{\boldsymbol{\delta}}_{\mathrm{n}}$ of the nonorthogonality misalignment is 360 arcsec for each of its components. The standard deviation of error in the initial estimate $\hat{\boldsymbol{\delta}}_{\mathrm{r}}$ of the rotational misalignment is 0.001 arcsec per axis. The covariance of the gyro axis misalignment vector is initialized according to (7), where $P_{\mathrm{r}} = \mathrm{cov}(\hat{\boldsymbol{\delta}}_{\mathrm{r}})$ and $P_{\mathrm{n}} = \mathrm{cov}(\hat{\boldsymbol{\delta}}_{\mathrm{n}})$. Thus, no rotational misalignment is estimated at the IMU so that the rotational misalignments estimated at the two star trackers are distinguishable in the measurements. The standard deviation of the process noise for the IMU misalignments was set to 0.0001 arcsec/hr$^{1/2}$ per axis. The standard deviation of the process noise for the star tracker misalignments was set to 0.12 arcsec/hr$^{1/2}$ per axis to permit tracking of the thermally-varying misalignments. Other filter parameters are described in a previous paper [2].

Star tracker and IMU telemetry from four calibration events (calibration maneuvers) were processed by the RADICAL attitude determination/calibration filter. These events occured on 2005300, 2005301, 2006216, and 2007262. The IMU contains two power supplies, PPSMA and PPSMB. The IMU was switched to PPSMB during the calibration event on 2005300, and then back to PPSMA prior to the next calibration event on 2005301. The IMU has remained on PPSMA since that time. Changing power supplies can significantly change the actual calibration parameters.

Although the Galileo star trackers output a measurement variance, it is quantized too much to be useful in the filter. The RADICAL filter can model the measurement error as a function of angular rate with a polynomial model, but insufficient information is available to determine appropriate coefficients of the model. Therefore a constant variance is used in the filter. Measurement residuals and parity residuals from processing the telemetry on 2007262 are shown in Figure 3. Also shown on the graphs are the $\pm 1\sigma$ bounds computed from the residual covariance matrix. It is seen in Figure 3a, as in previous results [2], that the measurement residuals are larger where the angular rate reaches 0.3 deg/sec during the calibration maneuver. This is due principally to distortion error and centroiding error. The centroiding error is aliased to a low spatial frequency because the angular rate of the maneuver causes the star image to move at nearly 1 pixel per sample, thus it is an unfortunate choice of angular rate for the calibration maneuver.

The parity residual is the component of the IMU measurements that lies in the null space of $\boldsymbol{W}$. The measurement update in the RADICAL filter includes the parity residual, as well as the star tracker measurements, so that the calibration parameters are fully observable [7, 8]. (The parity residual is also called a null-space measurement.) The parity residual in Figure 3b shows that it is zero mean and consistent with the $\pm 1\sigma$ bounds, although quantization error is evident. The measurement residuals and parity residuals show that the filter is performing well.



(a) STA measurement residuals **(b) Parity residuals**

Figure 3 Measurement residuals from filtering telemetry from 2006216.

Star Tracker Misalignment Estimation

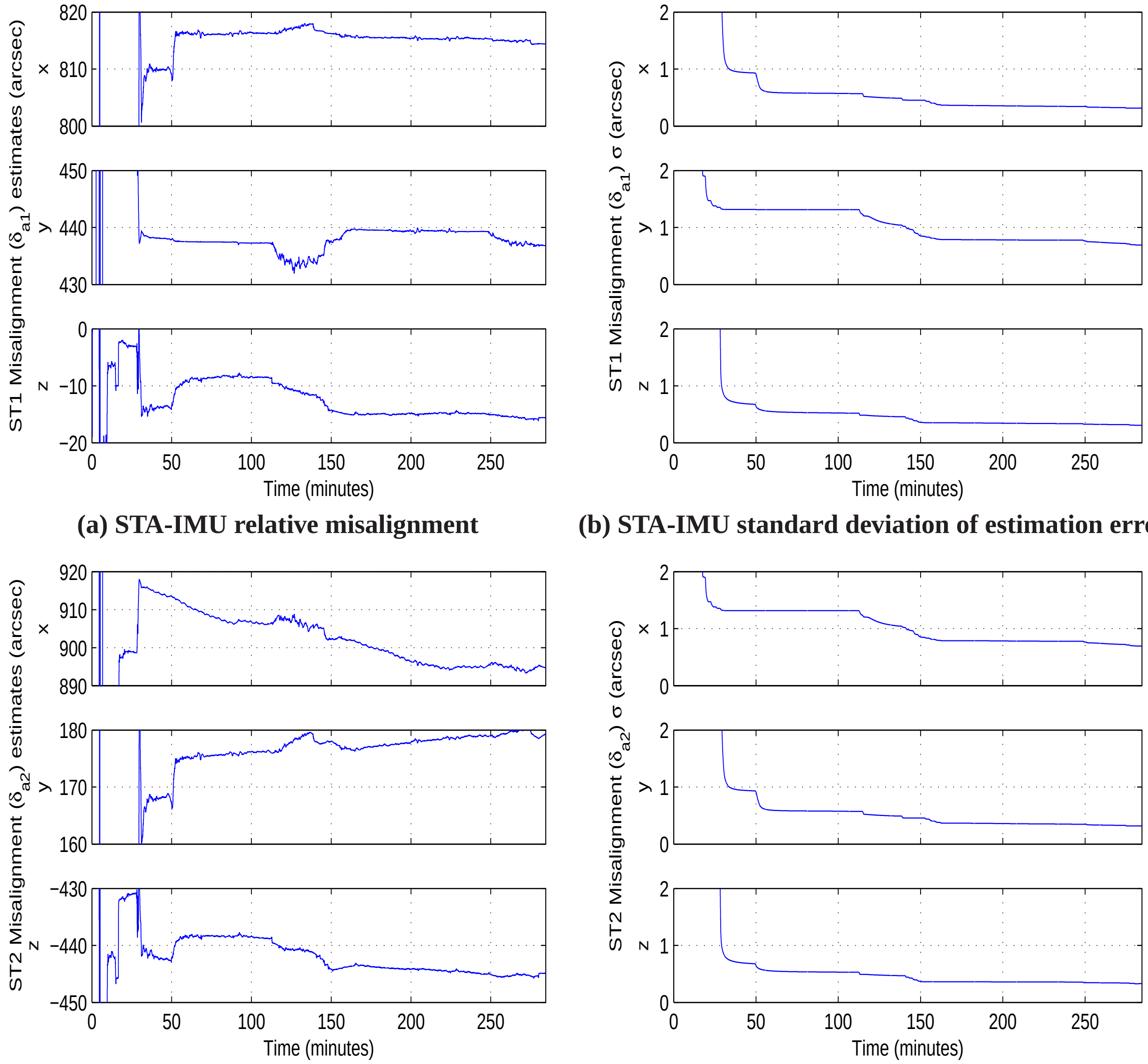
The final estimates of the star tracker misalignments from each calibration event are shown in Table 1. Since the IMU is the body reference sensor, the misalignment estimates of the star trackers produced by the filter are relative to the IMU. Plots of the misalignments relative to the IMU from 2006216 are shown in Figure 4. The variation in the misalignment is due to increasing baseplate temperatures of STA and STB, since the thermal control system is designed for single-tracker operation. The baseplate temperatures were shown previously [2]. It is seen in Figure 4c that the variation of the relative misalignment is due to changes in the alignment of STB relative to the IMU, whereas STA appears to be stable in Figure 4a. We can assume that the alignment of the IMU is stable since its baseplate temperature is constant.

The misalignment of STB relative to STA is computed by Eq. (8), and its covariance is computed by Eq. (10). These are shown in Figure 5, and are very close to previous results where STA was the body reference sensor [2].

STB was not turned on during the calibration maneuver that occured on 2007262. Therefore the relative misalignment between the star trackers cannot be estimated from the telemetry collected on 2007262. Plots of the misalignment of STA relative to the IMU from 2007262 are shown in Figure 6. This misalignment converges to a nearly constant value in each axis. A longer calibration maneuver, or one with larger angular rates or rotation angles, is needed for tighter convergence.

Table 1 Estimated star tracker misalignments and standard deviation, arcsec

Telemetry	x-axis		y-axis		z-axis	
	δ_1	σ_{δ_1}	δ_2	σ_{δ_2}	δ_3	σ_{δ_3}
STA alignment relative to IMU						
2005300	812.85	0.31	357.70	0.72	-12.22	0.32
2005301	808.86	0.31	431.84	0.70	-18.11	0.31
2006216	814.40	0.31	436.80	0.69	-15.59	0.31
2007262	797.07	0.55	395.89	1.20	-11.33	0.49
STB alignment relative to IMU						
2005300	966.02	0.72	176.15	0.32	-440.40	0.34
2005301	893.53	0.70	173.09	0.32	-444.43	0.33
2006216	894.72	0.69	179.32	0.32	-444.86	0.33
2007262	0.00		0.00		0.00	
STB alignment relative to STA						
2005300	1323.07	0.06	-637.00	0.06	-428.96	0.18
2005301	1324.72	0.06	-636.13	0.06	-426.96	0.18
2006216	1330.87	0.06	-635.43	0.06	-429.90	0.18



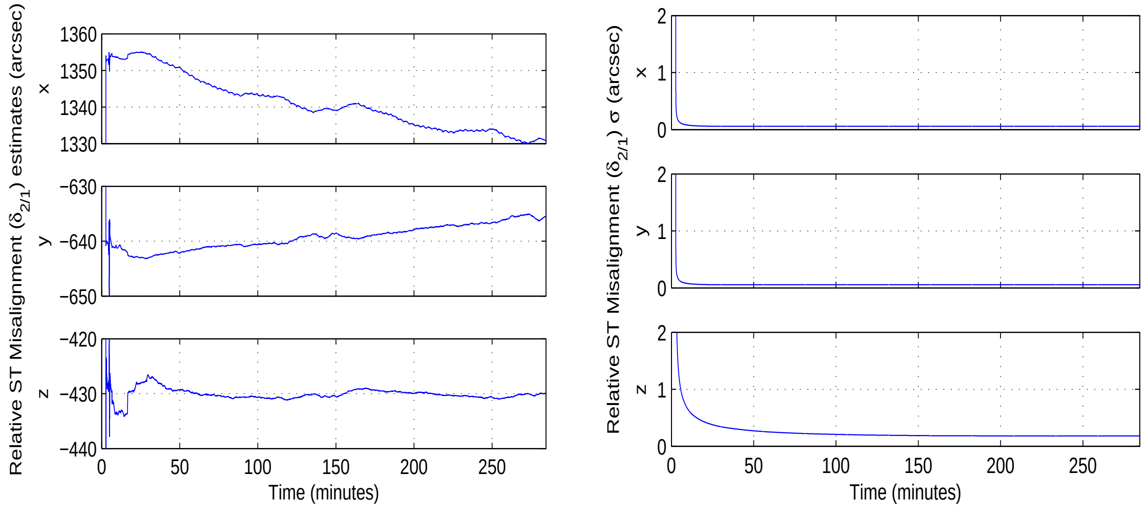
(a) STA-IMU relative misalignment

(b) STA-IMU standard deviation of estimation error

(c) STB-IMU relative misalignment

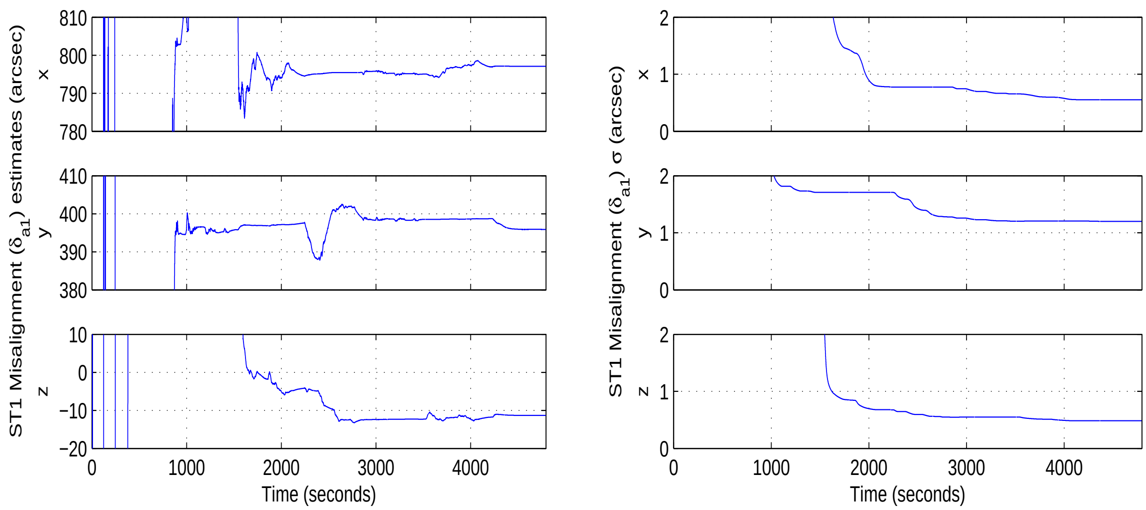
(d) STB-IMU standard deviation of estimation error

Figure 4 Star tracker misalignments and standard deviation from 2006216.



(a) **STB-STA relative misalignment** (b) **STB-STA standard deviation of estimation error**

Figure 5 Misalignment of STB relative to STA from 2006216.



(a) **STA-IMU relative misalignment** (b) **STA-IMU standard deviation of estimation error**

Figure 6 Misalignment of STA relative to IMU from 2007262.

Gyro Axis Misalignment Estimation

The final estimated gyro axis misalignments $\hat{\boldsymbol{\delta}}_{\boldsymbol{u}}$ and $\hat{\boldsymbol{\delta}}_{\boldsymbol{v}}$ from each calibration event are given in Tables 2 and 3. Because the rotational misalignment of the IMU is not estimated, $\hat{\boldsymbol{\delta}}_{\boldsymbol{u}}$ and $\hat{\boldsymbol{\delta}}_{\boldsymbol{v}}$ contain only the nonorthogonal misalignment of the gyro axes, which are typically much smaller than rotational misalignments. The non-orthogonal misalignment estimates and their covariance are computed from $\hat{\boldsymbol{\delta}}_{\boldsymbol{u}}$ and $\hat{\boldsymbol{\delta}}_{\boldsymbol{v}}$ by Eqs. (5) and (6). The non-orthogonal misalignment estimates and their standard deviations are plotted in Figures 7 and the final estimates are given in Table 4. The estimates converge to nearly constant values. However, the non-orthogonality estimates obtained from the four sets of telemetry show that the non-orthogonality changes over time. These estimates are not influenced by the rotational misalignment of the star trackers.

Table 2 Estimated gyro misalignments $\delta_{\boldsymbol{u}}$ and standard deviation, arcsec

Telemetry	Gyro A		Gyro B		Gyro C		Gyro D	
	$\delta_{\boldsymbol{u}_1}$	$\sigma_{\delta_{\boldsymbol{u}_1}}$	$\delta_{\boldsymbol{u}_2}$	$\sigma_{\delta_{\boldsymbol{u}_2}}$	$\delta_{\boldsymbol{u}_3}$	$\sigma_{\delta_{\boldsymbol{u}_3}}$	$\delta_{\boldsymbol{u}_4}$	$\sigma_{\delta_{\boldsymbol{u}_4}}$
2005300	−2.60	0.58	−46.58	0.71	13.01	0.58	36.11	0.71
2005301	4.96	0.56	6.64	0.69	4.21	0.56	−15.79	0.67
2006216	2.90	0.56	10.16	0.69	9.36	0.57	−22.40	0.68
2007262	−26.04	1.10	−4.57	1.13	27.36	1.10	3.27	1.13

Table 3 Estimated gyro misalignments $\delta_{\boldsymbol{v}}$ and standard deviation, arcsec

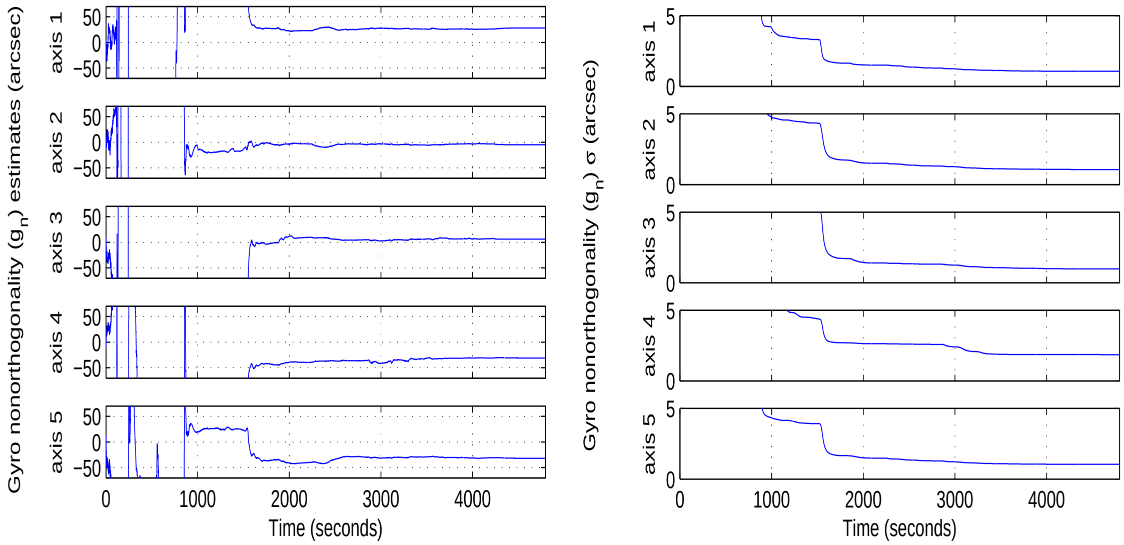
Telemetry	Gyro A		Gyro B		Gyro C		Gyro D	
	$\delta_{\boldsymbol{u}_1}$	$\sigma_{\delta_{\boldsymbol{u}_1}}$	$\delta_{\boldsymbol{u}_2}$	$\sigma_{\delta_{\boldsymbol{u}_2}}$	$\delta_{\boldsymbol{u}_3}$	$\sigma_{\delta_{\boldsymbol{u}_3}}$	$\delta_{\boldsymbol{u}_4}$	$\sigma_{\delta_{\boldsymbol{u}_4}}$
2005300	−0.11	0.57	25.19	0.50	−2.83	0.56	−34.34	0.51
2005301	8.17	0.56	−0.08	0.49	−11.85	0.55	−6.76	0.50
2006216	6.07	0.55	−3.18	0.49	−11.38	0.54	−5.59	0.50
2007262	4.11	0.84	26.99	0.82	−10.08	0.84	−22.53	0.83

Table 4 Estimated gyro nonorthogonal misalignments $\delta_{\boldsymbol{n}}$ and standard deviation, arcsec

Telemetry	$\delta_{\boldsymbol{n}_1}$	$\sigma_{\delta_{\boldsymbol{n}_1}}$	$\delta_{\boldsymbol{n}_2}$	$\sigma_{\delta_{\boldsymbol{n}_2}}$	$\delta_{\boldsymbol{n}_3}$	$\sigma_{\delta_{\boldsymbol{n}_3}}$	$\delta_{\boldsymbol{n}_4}$	$\sigma_{\delta_{\boldsymbol{n}_4}}$	$\delta_{\boldsymbol{n}_5}$	$\sigma_{\delta_{\boldsymbol{n}_5}}$
2005300	24.30	0.60	−17.35	0.71	41.68	0.60	13.67	1.10	−51.53	0.62
2005301	−14.44	0.59	8.88	0.70	−3.99	0.58	−15.77	1.06	−6.06	0.62
2006216	−16.35	0.59	6.32	0.69	−12.32	0.58	−20.51	1.06	−5.36	0.61
2007262	27.98	1.06	−5.07	1.06	6.11	0.99	−30.95	1.86	−31.76	1.04

CONCLUSION

The RADICAL filter was used to estimate calibration parameters using star tracker and IMU telemetry from the MESSENGER spacecraft. The IMU was made the body reference sensor by appropriately initializing the filter's covariance matrix to virtually eliminate a rotational misalignment vector from the eight misalignments of the four gyro axes. The misalignments of the star trackers are then relative to the IMU. The nonorthogonality misalignments of the gyro axes are extracted from



(a) Nonorthogonality misalignment estimate **(b) Standard deviation of estimation error**

Figure 7 Gyro axis nonorthogonality misalignment estimate and standard deviation from 2007262.

the gyro axis misalignments by a linear mapping. Results show that the estimated misalignments of the star trackers relative to the IMU are accurate to about 0.2 to 0.7 arcsec 1σ. Furthermore the estimated relative misalignment between the star trackers, computed from tracker misalignments relative to the IMU, is the same as the relative misalignment estimated when a star tracker is the body reference sensor and are accurate to about 0.1 arcsec 1σ.

Although the rotational misalignment could be explicitly eliminated from the filter, thus parameterizing the gyro axis misalignments with nonorthogonal misalignments and making the IMU the body reference sensor, this approach seems to complicate the calibration algorithm. An alternative approach [5] involving a constraint on the absolute misalignments may offer greater utility without greatly increasing the complexity of the calibration filter.

The results in this paper show that star tracker B on MESSENGER is subject to thermal deflection, and that star tracker A is stable with respect to the IMU. The thermo-mechanical design of the star tracker mountings should be reviewed to explain this finding. The radiator is designed for operation with only one star tracker on, so the baseplate temperatures of both star trackers rise when both star trackers are on. It should be emphasized that the attitude estimation performance on MESSENGER is adequate. This work serves to gain a greater understanding of the design and potential performance of the system and at the same time to demonstrate the capabilities and performance of the RADICAL attitude determination/calibration filter.

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